

The Design of Statistical Studies (Ch 1-2)

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Common Experimental Designs

Completely Randomized Design

Factorial Design

Randomized Complete Block Design

Common
Experimental
Designs

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Factorial Design
Randomized Complete
Block Design

Simple Random
Sampling

Simple Random Sampling

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Randomization
without blocks
Randomization with
Blocks

Randomization

Randomization without blocks

Randomization with Blocks

Completely Randomized Design

▶ Completely Randomized Design

- ▶ an experimental design with one treatment variable and no blocking variables.
- ▶ Sample units are randomly assigned to treatment levels.

▶ Example: metallurgy

- ▶ Test the effect of different additives on the corrosion rate of steel.
- ▶ Sample: 12 pieces of raw iron
- ▶ Treatment: additive (A, B, or C).
- ▶ Treatment groups: A (units 1-4), B (units 5-8), and C (units 9-12)

| Sample unit | Additive | Sample unit | Additive |
|-------------|----------|-------------|----------|
| 1 | A | 7 | B |
| 2 | A | 8 | B |
| 3 | A | 9 | C |
| 4 | A | 10 | C |
| 5 | B | 11 | C |
| 6 | B | 12 | C |

► Factorial Design

- an experimental design with multiple treatment variable (as factors) and no blocking variables.
- Each sample unit is randomly assigned to a combination of treatment levels.

► Example: metallurgy: a 3×2 factorial version

- Treatment 1: additive (A, B, or C).
- Treatment 2: temperature (high or low)
- Treatment groups: A high (units 1-2), A low (units 3-4), B high (units 5-6), B low (units 7-8), C high (units 9-10), C low (units 11-12),

| Unit | Additive | Temp | Unit | Additive | Temp |
|------|----------|------|------|----------|------|
| 1 | A | high | 7 | B | low |
| 2 | A | high | 8 | B | low |
| 3 | A | low | 9 | C | high |
| 4 | A | low | 10 | C | high |
| 5 | B | high | 11 | C | low |
| 6 | B | high | 12 | C | low |

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Metallurgy example: a 2^3 factorial version

- ▶ Sample: 16 pieces of iron.
- ▶ Treatments:
 - ▶ Treatment 1: additive (A or B)
 - ▶ Treatment 2: temperature (high or low)
 - ▶ Treatment 3: smelting time (long or short)
- ▶ Treatment groups: A high long (units 1-2) A high short (units 3-4), ..., B low short (units 11-12).

| Unit | Add | Temp | Smelt | Unit | Add | Temp | Smelt |
|------|-----|------|-------|------|-----|------|-------|
| 1 | A | high | long | 9 | B | high | long |
| 2 | A | high | long | 10 | B | high | long |
| 3 | A | high | short | 11 | B | high | short |
| 4 | A | high | short | 12 | B | high | short |
| 5 | A | low | long | 13 | B | low | long |
| 6 | A | low | long | 14 | B | low | long |
| 7 | A | low | short | 15 | B | low | short |
| 8 | A | low | short | 16 | B | low | short |

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Randomized Complete Block Design

► Randomized Complete Block Design

- an experimental design with one or more treatment variable and at least one blocking variable.
- Within each block separately, sample units are assigned to treatment groups

► Example: metallurgy

- Treatment: additive (A, B, or C).
- Blocking variable: pig iron supplier (Amset or Miller and Co.)

| Unit | Supplier | Add | Unit | Supplier | Add |
|------|----------|-----|------|----------|-----|
| 1 | Amset | A | 7 | Miller | A |
| 2 | Amset | A | 8 | Miller | A |
| 3 | Amset | B | 9 | Miller | B |
| 4 | Amset | B | 10 | Miller | B |
| 5 | Amset | C | 11 | Miller | C |
| 6 | Amset | C | 12 | Miller | C |

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Simple Random Sampling

- ▶ **Simple Random Sampling:** drawing a sample of n units from a finite population of N units such that every possible n -sized subset of the population has an equal chance of being selected.
- ▶ Use either a computerized random number generator or a [table of random digits](#).

Random Digits

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 | 13684 | 66024 | 91410 | 51351 | 22772 |
| 30156 | 90519 | 95785 | 47544 | 66735 | 35754 | 11088 | 67310 | 19720 | 08379 |
| 59069 | 01722 | 53338 | 41942 | 65118 | 71236 | 01932 | 70343 | 25812 | 62275 |
| 54107 | 58081 | 82470 | 59407 | 13475 | 95872 | 16268 | 78436 | 39251 | 64247 |
| 99681 | 81295 | 06315 | 28212 | 45029 | 57701 | 96327 | 85436 | 33614 | 29070 |

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Steps of Simple Random Sampling

1. Let M be the number of digits in the number $N - 1$, where N is the population size. (If $N = 1000$ then $M = 3$ digits.)
2. Give each member of the population an M -digit index, i (say, $i = 000, 001, \dots, 999$)
3. Move through the table of random digits from left to right, top to bottom, selecting population members for the sample when you encounter their indices (ignoring indices that have already been chosen) until you have selected n units for the sample.

Random Digits

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 | 13684 | 66024 | 91410 | 51351 | 22772 |
| 30156 | 90519 | 95785 | 47544 | 66735 | 35754 | 11088 | 67310 | 19720 | 08379 |
| 59069 | 01722 | 53338 | 41942 | 65118 | 71236 | 01932 | 70343 | 25812 | 62275 |
| 54107 | 58081 | 82470 | 59407 | 13475 | 95872 | 16268 | 78436 | 39251 | 64247 |
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Your turn: metallurgy

Using the table of random digits below, take a simple random sample of 12 units of pig iron out of a shipment of 90 units.

| | | | | |
|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 |
| 30156 | 90519 | 95785 | 47544 | 66735 |
| 59069 | 01722 | 53338 | 41942 | 65118 |
| 54107 | 58081 | 82470 | 59407 | 13475 |
| 99681 | 81295 | 06315 | 28212 | 45029 |

Your turn: metallurgy

Solution:

- ▶ Indexed the members of the population from 00 to 89.
- ▶ Selected units 12, 15, 61, 44, 5, 9, 11, 34, 46, 45, 65, and 33 for the sample.

| | | | | |
|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 |
| 30156 | 90519 | 95785 | 47544 | 66735 |
| 59069 | 01722 | 53338 | 41942 | 65118 |
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- ▶ **Randomization:** assigning sample units to treatment groups in an experiment such that every set of assignments is equally likely.
- ▶ Steps to randomize n sample units to t treatment groups, each of size s ($n = ts$):
 1. Use the table of random digits to select s units for treatment group 1 from the experimental sample of n units.
 2. Continuing from your last spot in the table, select s units for treatment group 2 from the remaining $n - s$ units in the experimental sample.
 3. Continue this process until you have selected $t - 1$ treatment groups. The remaining units will belong to the last treatment group.

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Your turn: metallurgy

Randomize our experimental sample of 12 units of pig iron to three treatment groups (for additives A, B, and C).

| | | | | |
|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 |
| 30156 | 90519 | 95785 | 47544 | 66735 |
| 59069 | 01722 | 53338 | 41942 | 65118 |
| 54107 | 58081 | 82470 | 59407 | 13475 |
| 99681 | 81295 | 06315 | 28212 | 45029 |

Your turn: metallurgy

Solution:

- ▶ Units 05, 09, 11, and 01 for group A (blue).
- ▶ Units 06, 07, 08, and 02 for group B (green).
- ▶ Units 03, 04, 10, and 00 for group C (leftover).

| | | | | |
|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 |
| 30156 | 90519 | 95785 | 47544 | 66735 |
| 59069 | 01722 | 53338 | 41942 | 65118 |
| 54107 | 58081 | 82470 | 59407 | 13475 |
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For randomization in factorial studies, know your treatment groups.

- ▶ Example: metallurgy: a 3×2 factorial version

- ▶ Sample: $n = 12$ units
- ▶ Treatment 1: additive (A, B, or C).
- ▶ Treatment 2: temperature (high or low).

1. How many treatment groups do we have?
2. How many units of the experimental sample should I randomize to each treatment group?

Know your treatment groups: answers

1. $3 \times 2 = 6$ treatment groups.
2. Each treatment group has $12/6 = 2$ units of pig iron.

Randomization with blocks

- ▶ Randomize units to treatments *within each block*.

| Unit | Supplier | Add | Unit | Supplier | Add |
|------|----------|-----|------|----------|-----|
| 1 | Amset | A | 7 | Miller | A |
| 2 | Amset | A | 8 | Miller | A |
| 3 | Amset | B | 9 | Miller | B |
| 4 | Amset | B | 10 | Miller | B |
| 5 | Amset | C | 11 | Miller | C |
| 6 | Amset | C | 12 | Miller | C |

- ▶ For the metallurgy block design:
 - ▶ Randomize all *Amset units* to treatments A, B, and C
 - ▶ Then, picking up where you left off in the table of random digits, randomize all *Miller units* to treatments A, B, and C.

Your turn: metallurgy block design

- ▶ Given:
 - ▶ 2 blocks (Amset and Miller).
 - ▶ 3 treatment levels (A, B, and C).
- ▶ Randomize the 12 units of pig iron to treatment groups

| | | | | |
|-------|-------|-------|-------|-------|
| 12159 | 66144 | 05091 | 13446 | 45653 |
| 30156 | 90519 | 95785 | 47544 | 66735 |
| 59069 | 01722 | 53338 | 41942 | 65118 |
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The Amset Block

- ▶ Index the 6 Amset units of pig iron from 0 to 5.
- ▶ Using the table of random digits, select:
 - ▶ Units 1 and 2 for group A (blue).
 - ▶ Units 5 and 4 for group B (green).
 - ▶ Units 0 and 3 for group C (leftover).

| | | | | |
|-------|-------|-------|-------|-------|
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The Miller Block

- ▶ Index the 6 Miller units of pig iron from 0 to 5.
- ▶ Using the table of random digits, select:
 - ▶ Units 4 and 0 for group A (orange).
 - ▶ Units 5 and 1 for group B (red).
 - ▶ Units 2 and 3 for group C (leftover).

| | | | | |
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| 59069 | 01722 | 53338 | 41942 | 65118 |
| 54107 | 58081 | 82470 | 59407 | 13475 |
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