

STAT 305 D Homework 7

Due March 28, 2013 at 12:40 PM in class

1. Vardeman and Jobe chapter 5 section 4 problem 1 (page 300)

Explain in qualitative terms what it means for two random variables X and Y to be independent. What advantage is there when X and Y can be described as independent?

2. Vardeman and Jobe chapter 5 section 4 problem 2 (page 300)

Quality audit records are kept on numbers of major and minor failures of circuit packs during burn-in of large electronic switching devices. They indicate that for a device of this type, the random variables

X = the number of major failures

and

Y = the number of minor failures

can be described at least approximately by the accompanying joint distribution.

$y \backslash x$	0	1	2
0	.15	.05	.01
1	.10	.08	.01
2	.10	.14	.02
3	.10	.08	.03
4	.05	.05	.03

- (a) Find the marginal probability functions for both X and Y — $f_X(x)$ and $f_Y(y)$.
- (b) Are X and Y independent? Explain.
- (c) Find the mean and variance of X — EX and $\text{Var } X$.
- (d) Find the mean and variance of Y — EY and $\text{Var } Y$.
- (e) Find the conditional probability function for Y , given that $X = 0$ —i.e., that there are no major circuit pack failures. (That is, find $f_{Y|X}(y | 0)$.)

3.

- a. $E(X + 2Y)$, where $X \sim \text{Binomial}(n = 3, p = .7)$ and is independent of $Y \sim \text{Geometric}(p = 0.3)$
- b. $\text{Var}(X + 2Y)$, where $X \sim \text{Binomial}(n = 3, p = .7)$ and is independent of $Y \sim \text{Geometric}(p = 0.3)$
- c. $E(3X + 5Y - 8Z)$, where X, Y , and Z are independent, $E(X) = 4$, $E(Y) = 3$, $E(Z) = 10$
- d. $\text{Var}(3X + 5Y - 8Z)$, where X, Y , and Z are independent, $\text{Var}(X) = 9$, $\text{Var}(Y) = 16$, $\text{Var}(Z) = 2$
- e. $E(\bar{X})$, where X_1, X_2, \dots, X_n are iid with mean 0 and variance 2.
- f. $P(\bar{X} \leq 6)$, where $X_1, X_2, \dots, X_{40} \sim \text{iid } N(5, 2)$
- g. $P(\bar{X} > 11)$, where $X_1, X_2, \dots, X_{20} \sim \text{iid } N(3, 150)$
- h. $P(|\bar{X} - 3| > 1)$, where $X_1, X_2, \dots, X_{10} \sim \text{iid } N(3, 18)$

4.

- a. $P(\bar{X} \leq 23)$, where $X_1, X_2, \dots, X_{40} \sim \text{iid Binomial}$ with mean 20 and variance 100.
- b. $P(\bar{X} > 1)$, where $X_1, X_2, \dots, X_{86} \sim \text{iid Scaled Exponential}$ with mean 0 and variance 28.
- c. $P(|\bar{X} + 8| > 12)$, where $X_1, X_2, \dots, X_{70} \sim \text{iid Weibull}$ with mean 3 and variance 37.

5.

Students are going to measure Young's Modulus for copper by measuring the elongation of a piece of copper wire under a tensile force. For a cylindrical wire of diameter D subjected to a tensile force F , if the initial length (length before applying the force) is L_0 and final length is L_1 , Young's Modulus for the material in question is

$$Y = \frac{4FL_0}{\pi D^2(L_1 - L_0)}$$

The test and measuring equipment used in a particular lab are characterized by the standard deviations

$$\begin{aligned}\sigma_F &\approx 10 \text{ lb} & \sigma_D &\approx .001 \text{ in.} \\ \sigma_{L_0} &= \sigma_{L_1} = .01 \text{ in.}\end{aligned}$$

and in the setup employed, $F \approx 300 \text{ lb}$, $D \approx .050 \text{ in.}$, $L_0 \approx 10.00 \text{ in.}$, and $L_1 \approx 10.10 \text{ in.}$

- (a) Treating the measured force, diameter, and lengths as independent variables with the preceding means and standard deviations, find an approximate standard deviation to attach to an experimentally derived value of Y . (Partial derivatives of Y at the nominal values of F , D , L_0 , and L_1 are approximately $\frac{\partial Y}{\partial F} \approx 5.09 \times 10^4$, $\frac{\partial Y}{\partial D} \approx -6.11 \times 10^8$, $\frac{\partial Y}{\partial L_0} \approx 1.54 \times 10^8$, and $\frac{\partial Y}{\partial L_1} \approx -1.53 \times 10^8$ in the appropriate units.)
- (b) Uncertainty in which of the variables is the biggest contributor to uncertainty in Y ?

6. Weekly feedback. You get full credit as long as you write something.
 - a. Is there any aspect of the subject matter that you currently struggle with? If so, what specifically do you find difficult or confusing? The more detailed you are, the better I can help you.
 - b. Do you have any questions or concerns about the material, class logistics, or anything else? If so, fire away.