STAT 305 D Homework 6

Due March 7, 2013 at 12:40 PM in class

Remember: μ denotes $E(X)$, σ^2 denotes $\mathrm{Var}(X)$,m and σ denotes $\mathrm{SD}(X)$.

1.

The article "Modeling Sediment and Water Column Interactions for Hydrophobic Pollutants" (*Water Research,* 1984: 1169–1174) suggests the uniform distribution on the interval (7.5, 20) as a model for depth (cm) of the bioturbation layer in sediment in a certain region.

a. What are the mean and variance of depth?

b. What is the cdf of depth?

c. What is the probability that observed depth is at most 10? Between 10 and 15?

d. What is the probability that the observed depth is within 1 standard deviation of the mean value? Within 2 standard deviations?

2.

Let X be the total medical expenses (in 1000s of dollars) incurred by a particular individual during a given year. Although X is a discrete random variable, suppose its distribution is quite well approximated by a continuous distribution with pdf $f(x) = k(1 + x/2.5)^{-7}$ for $x \geq 0$.

a. What is the value of k?

b. Graph the pdf of X.

c. What are the expected value and standard deviation of total medical expenses?

d. This individual is covered by an insurance plan that entails a \$500 deductible provision (so the first \$500 worth of expenses are paid by the individual). Then the plan will pay 80% of any additional expenses exceeding \$500, and the maximum payment by the individual (including the deductible amount) is \$2500. Let Y denote the amount of this individual's medical expenses paid by the insurance company. What is the expected value of Y?

[*Hint:* First figure out what value of X corresponds to the maximum out-of-pocket expense of \$2500. Then write an expression for Y as a function of X (which involves several different pieces) and calculate the expected value of this function.]

3. For X with a continuous distribution specified by the probability density

$$f(x) = \begin{cases} .5x & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

find $P[X < 1.0]$ and find the mean, EX.

4. Suppose that Z is a standard normal random variable. Evaluate the following probabilities involving Z:

(a) $P[Z < -.62]$ (b) $P[Z > 1.06]$
(c) $P[-.37 < Z < .51]$ (d) $P[|Z| \leq .47]$
(e) $P[|Z| > .93]$ (f) $P[-3.0 < Z < 3.0]$

Now find numbers # such that the following statements involving Z are true:

(g) $P[Z \leq \#] = .90$ (h) $P[|Z| < \#] = .90$
(i) $P[|Z| > \#] = .03$

5. Find the following:

a.

a. $P(X \leq 2)$, where $X \sim N(5, 2)$

b.

b. $P(|X| \leq 5)$, where $X \sim N(0, 15)$

c.

c. $P(T \geq 1.9432)$, where $T \sim t_6$

d.

$P(|T| \leq 1.372)$, where $T \sim t_{10}$.

e.

$P(X > 7.779)$, where $X \sim \chi^2_4$

f.

$P(X \leq 11.3449)$, where $X \sim \chi^2_3$

g.

$P(F \leq 30.8165)$, where $F \sim F_{2,3}$

h.

$P(F > 4.8759)$, where $F \sim F_{7,5}$

6. Review the notation explained on the last slide of the Feb 28 lecture. Then, find the following.

   a. $z_{0.8}$
   b. $t_{3,0.9}$
   c. $\chi^2_{2,0.95}$
   d. $F_{4,2,0.99}$

7. Let $S \sim \text{SquigglyJoe}(5, 3, \xi, \xi_\xi, \xi^{\xi^\xi}_{\xi_\xi}, \xi^{\xi^{\xi_\xi}}_{\xi_{\xi_\xi}})$, where $E(S) = 42$ and $Var(S) = 101$. Find the following:

   a. E(5 S + 7)
   b. Var(10S -2)

8. Weekly feedback. You get full credit as long as you write something.

   a. Is there any aspect of the subject matter that you currently struggle with? If so, what specifically do you find difficult or confusing? The more detailed you are, the better I can help you.
   b. Do you have any questions or concerns about the material, class logistics, or anything else? If so, fire away.